SAMPLE P.1/7

SET 1

MATHEMATICS PAPER 1
Section B Mock
Question-Answer Book

This paper must be answered in English

You are advised to finish section B within 35 minutes

INSTRUCTIONS

1. Attempt ALL questions in Section B. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

2. Graph paper and supplementary answer sheets will be supplied on request. Write your Name on each sheet, and fasten them with string INSIDE this book.

3. Unless otherwise specified, all working must be clearly shown.

4. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

5. The diagrams in this paper are not necessarily drawn to scale.

SAMPLE P.2/7

FORMULAS FOR REFERENCE

SPHERE	Surface area	$4\pi r^2$
	Volume	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	$2\pi rh$
	Volume	$\pi r^2 h$
CONE	Area of curved surface	πrl
	Volume	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	base area $\times$ height
PYRAMID	Volume	$\frac{1}{3}$ $\times$ base area $\times$ height

1. (a) In an arithmetic sequence, the first term is 1 and the common difference is

(i) Find the general term of the sequence.

(ii) Is it a geometric sequence? Explain your answer.

(iii) Find the sum $S(n)$ of the first n terms of the sequence in terms of n.

(4 marks)

(b) The $S(n)$ obtained in (a)(iii) is the general term of the sequence of triangular numbers.

(i) Calculate $S(15) + S(16)$.

(ii) Prove that the sum of two consecutive triangular numbers $S(n)$ and $S(n + 1)$ must be a square number.

(4 marks)

(c) Mr Chan is a designer for the window decoration of a plaza. He divides 484 decorations into two groups P and Q. The numbers of decorations in P and Q are two triangular numbers and there are more decorations in P. P is then evenly divided into a certain number of small groups, where the numbers of decorations in each group is more than 1. How does he divide these decorations?

(3 marks)

2. (a) In Figure (a), OPQ is an acute-angled triangle and OQP' is an equilateral triangle. Circle $ORQP'$ is the circumcircle of $\Delta OQP'$. $PRR'P'$ is a straight line and $RR' = OR$.

(i) Prove that $\Delta ORR'$ is an equilateral triangle.

(ii) Prove that $QR = P'R'$.

(iii) R'' is the mid-point of OQ. Suppose O, P and Q are three places in a city. A company's headquarter is proposed to locate in this region. In order to enhance the efficiency and reduce the cost, the sum of distances of the three places from the headquarter must be minimized. On which point, R or R'', should the headquarter be located? Explain your answer.

(6 marks)

(6 marks)

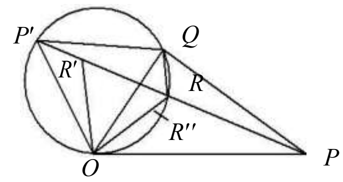

Figure (a)

(b) Consider the situation of ΔOPQ below in Figure (b) and introduce a rectangular coordinate system with O as the origin such that the coordinates of Q are $(2\ ,\ 2\sqrt{3})$ and the point $P(4\ ,\ 0)$ lies on the x-axis.

(i) Find $\angle POQ$.

(ii) A is a point on the minor arc OQ of the circle. Find the minimum value of the sum of distances of the vertices from A.

(5 marks)

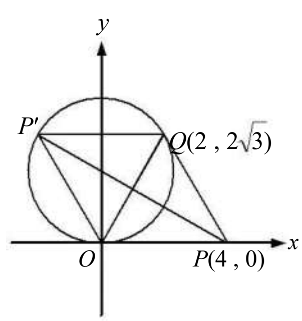

Figure (b)

3. In Figure 1, VP is a transmitter which is placed vertically on the ground. Q and R are due east and due south of P respectively. It is given that $VP = 2$ m, $VQ = 8$ m, $VR = 2\sqrt{6}$ m.

(a) (i) Find QR.

(ii) Use Heron's formula to find the area of ΔVQR, leaving the radical sign in your answer.

(iii) What is the relationship between the sum of the squares of areas of ΔVPQ, ΔVPR, ΔPQR and the square of area of ΔVQR?

(6 marks)

(b) It is given that the bearing of S from R is N45°E and $PS = \sqrt{10}$ m.

(i) Find RS and VS.

(ii) Is the relationship between the sum of the squares of areas of ΔVPS, ΔVPR, ΔPRS and the square of area of ΔVRS the same as the relationship in (a)(iii)? Explain your answer.

(5 marks)

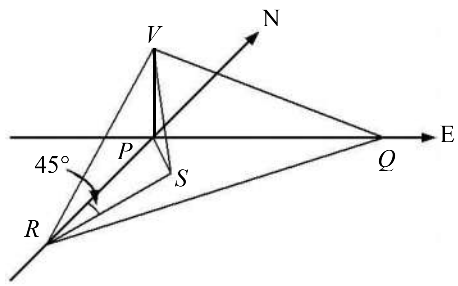

Figure 1

ANSWER KEY:

3. (a) (i) $PQ = \sqrt{8^2 - 2^2}$ m
$= \sqrt{60}$ m
$= 2\sqrt{15}$ m
$PR = \sqrt{(2\sqrt{6})^2 - 2^2}$ m
$= \sqrt{20}$ m
$= 2\sqrt{5}$ m
$QR = \sqrt{(2\sqrt{5})^2 + (2\sqrt{15})^2}$ m
$= \sqrt{80}$ m
$= \underline{\underline{4\sqrt{5}\text{ m}}}$ 1A

(ii) Let $s = \dfrac{8 + 2\sqrt{6} + 4\sqrt{5}}{2}$ m
$= (4 + \sqrt{6} + 2\sqrt{5})$ m 1A
Area of ΔVQR 1M

$= \sqrt{(4+\sqrt{6}+2\sqrt{5})[(4+\sqrt{6}+2\sqrt{5})-8][(4+\sqrt{6}+2\sqrt{5})-2\sqrt{6}][(4+\sqrt{6}+2\sqrt{5})-4\sqrt{5}]}$ m^2
$= \sqrt{[(4+\sqrt{6})+2\sqrt{5}][(4+\sqrt{6})-2\sqrt{5}][2\sqrt{5}+(4-\sqrt{6})][2\sqrt{5}-(4-\sqrt{6})]}$ m^2
$= \sqrt{(16+8\sqrt{6}+6-20)(20-16+8\sqrt{6}-6)}$ m^2
$= \sqrt{(2+8\sqrt{6})(8\sqrt{6}-2)}$ m^2 1A
$= \underline{\underline{\sqrt{380}\text{ m}^2}}$

(iii) (Area of ΔVPQ)^2 + (area of ΔVPR)^2 + (area of ΔPQR)^2
$= \left\{\left[\frac{1}{2}(2)(2\sqrt{15})\right]^2 + \left[\frac{1}{2}(2)(2\sqrt{5})\right]^2 + \left[\frac{1}{2}(2\sqrt{15})(2\sqrt{5})\right]^2\right\}$ m^4 1M
$= 380$ m^4
(Area of ΔVQR)^2 □ $(\sqrt{380})^2$ m^4
□□380 m^4

∴ The sum of the squares of areas of ΔVPQ, ΔVPR, ΔPQR is the same as the square of area of ΔVQR. 1A
(6)

(b) (i) Let RS □□ x m.
$PS^2 = PR^2 + RS^2 - 2(PR)(RS)\cos\angle PRS$
$(\sqrt{10})^2 = (2\sqrt{5})^2 + x^2 - 2(2\sqrt{5})(x)\cos 45°$
$10 = 20 + x^2 - 4\sqrt{5}\left(\frac{1}{\sqrt{2}}\right)x$

$x^2 - 2\sqrt{10}x + 10 = 0$
$(x - \sqrt{10})^2 = 0$
$x = \sqrt{10}$
$RS = \underline{\underline{\sqrt{10}\text{ m}}}$ 1A

$$VS = \sqrt{VP^2 + PS^2}$$
$$= \sqrt{(2)^2 + (\sqrt{10})^2} \text{ m}$$
$$= \underline{\underline{\sqrt{14} \text{ m}}}$$ 1A

(ii) (Area of ΔVPS)^2 + (area of ΔVPR)^2 + (area of ΔPRS)^2

$$= \left\{\left[\frac{1}{2}(2)(\sqrt{10})\right]^2 + \left[\frac{1}{2}(2)(2\sqrt{5})\right]^2 + \left[\frac{1}{2}(2\sqrt{5})(\sqrt{10})\sin 45°\right]^2\right\} \text{ m}^4$$
$$= 55 \text{ m}^4$$

$$\cos\angle RVS = \frac{(2\sqrt{6})^2 + (\sqrt{14})^2 - (\sqrt{10})^2}{2(2\sqrt{6})(\sqrt{14})}$$
$$= \frac{\sqrt{21}}{6}$$ 1M

$$\cos^2\angle RVS + \sin^2\angle RVS = 1$$
$$\sin\angle RVS = \sqrt{1 - \left(\frac{\sqrt{21}}{6}\right)^2} = \frac{\sqrt{15}}{6}$$

$$(\text{Area of } \Delta VRS)^2 = \left[\frac{1}{2}(2\sqrt{6})(\sqrt{14})\left(\frac{\sqrt{15}}{6}\right)\right]^2 \text{ m}^4$$ 1M
$$= 35 \text{ m}^4$$

$\therefore$ The relationship between the sum of the squares of areas of ΔVPS, ΔVPR, ΔPRS and the square of area of ΔVRS is different from the relationship in (a)(iii). 1A

(5)

End of Paper